Flow Past Nonconical Wings with Separation

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Introduction

IN this Note, an extension of the Brown and Michael[1] method to evaluate the effect of flow separation on wings of nonconical planforms is presented. Here, it is assumed that the spiral vortex sheets that separate from the leading edges could be replaced by two isolated vortices at the cores of the vortex sheets. The vortex strength increases in the downstream direction, and the increase in vorticity is accomplished in this model by a feeding sheet of vorticity in order to satisfy Kelvin's theorem. Although this approximation leads to higher values for lift coefficient than those obtained from experiments, it gives a first approximation for the evaluation of the vortex lift.

Governing Equations

A flat-plate wing of planform $s=F(x)$ (where s is the semispan of the wing), kept at an angle of attack α to the freestream, is considered (Fig. 1). The presence of leading-edge separation is assumed. V represents the freestream velocity. The velocity potential of the flow is given by

$$\Phi = V\cdot x + \phi \tag{1}$$

where the first term is due to the component of the freestream parallel to the plane of the wing ($\cos\alpha \approx 1$), and the second term is the sum of the potential due to the component of the freestream normal to the wing plane and the disturbance potential. Assuming the wing to be slender, the equation of motion reduces to

$$\phi_{zz} + \phi_{yy} = 0 \tag{2}$$

where the coordinates are as given in Fig. 1. Thus the problem reduces to solving Eq. (2) in the cross-flow plane (yz plane).

Boundary Conditions

The conditions on the wing are as follows:
1) The normal velocity is zero.
2) The velocity is finite at the leading edge.
3) The conditions in the field are that a) the disturbances vanish at infinity, and b) the fluid pressure is continuous.

Using the transformation

$$Z^{*2} = Z^2 - s^2 \tag{3}$$

the wing in the Z plane is transformed to a vertical slit in the Z^* plane. Thus, the normal velocity condition on the wing is satisfied automatically.

From the Z^* plane, the complex velocity potential can be written as

$$W(Z^*) = \frac{-i\Gamma}{2\pi}\log\frac{Z^* - Z^*_v}{Z^* + \bar{Z}^*_v} - i\alpha V Z^* \tag{4}$$

where the two isolated vortices of strength Γ and $-\Gamma$ are located at Z^*_v and $-\bar{Z}^*_v$, respectively.

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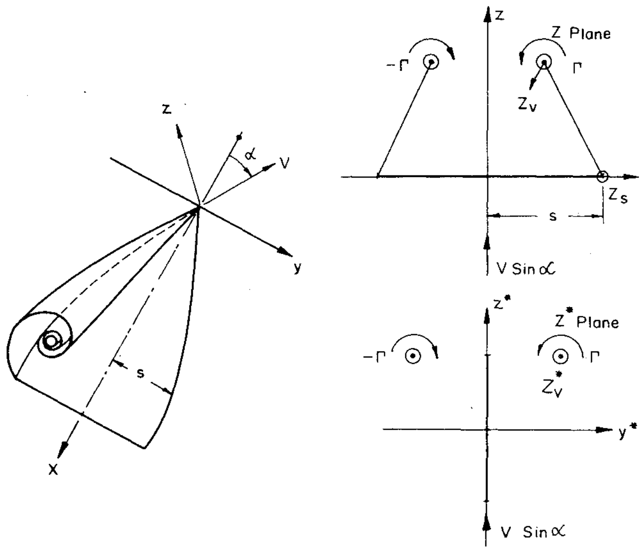

Fig. 1 Flow model for nonconical wings.

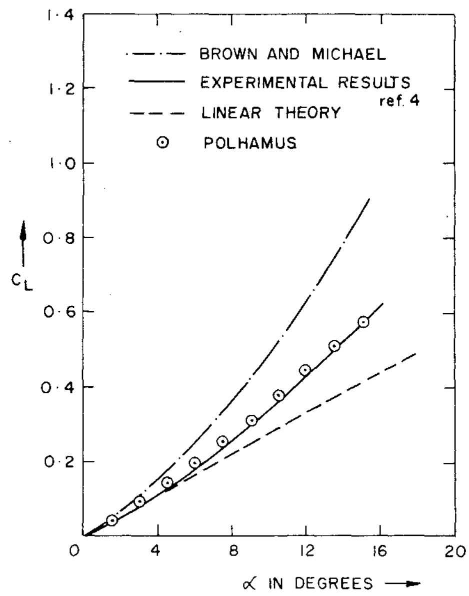

Fig. 2 Nonconical wing C_L vs α.

The second boundary condition reduces to $dW/dZ^* = 0$ at $Z^* = 0$. Substituting for W from Eq. (4) and rearranging, we get

$$\Gamma = 2\pi\alpha V\frac{Z^*_v\cdot\bar{Z}^*_v}{Z^*_v + \bar{Z}^*_v} \tag{5}$$

From the expression for W, it can be seen that boundary condition 3a is satisfied automatically.

It is impossible to satisfy boundary condition 3b with the assumed model. The difficulty lies in the presence of the feeding vortex sheet, across which there is a pressure discontinuity, but, since the assumed vortex system represents the true spiral only at large distances, it is to be expected that in the small regions near the system violation of natural conditions occurs. Hence, the boundary condition 3b is simplified by assuming that the total force on the vortex and cut is equal to zero. This leads to[2]

$$\frac{dZ_v}{dx} + \frac{Z_v - Z_s}{\Gamma}\frac{d\Gamma}{dx} = \frac{v}{V} \tag{6}$$

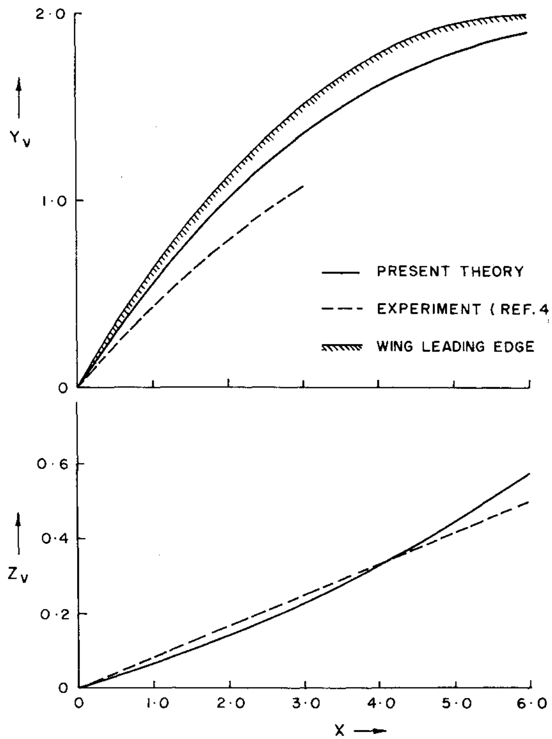

Fig. 3 y_v **and** Z_v **vs** x **(**$\alpha = 15.3$ **deg).**

where v can be obtained from

$$\bar{v} = \lim_{Z \to Z_v} \left(\frac{dW}{dZ} + \frac{i\Gamma}{2\pi} \frac{1}{Z - Z_v} \right) \tag{7}$$

Substituting for W and Γ from Eqs. (4) and (5) in Eq. (7), it reduces to

$$\bar{v} = -i\alpha V \frac{Z_v}{Z^*_v} + \frac{i\Gamma}{2\pi} \frac{1}{Z^*_v + \bar{Z}^*_v} \frac{Z_v}{Z^*_v} + \frac{i\Gamma}{2\pi} \frac{s^2}{2Z_v Z^{*2}_v} \tag{8}$$

Substituting for Γ and v in Eq. (6) and separating it into real and imaginary parts, we get

$$A\frac{dy_v}{dx} + B\frac{dZ_v}{dx} = \text{real}\left(\frac{v}{V}\right) \tag{9}$$

$$C\frac{dy_v}{dx} + D\frac{dZ_v}{dx} = \text{imag}\left(\frac{v}{V}\right) \tag{10}$$

where $y_v + iZ_v = Z_v$, and A, B, C, and D are functions of Z_v and x. Hence, if Z_v, the position of the isolated vortex, at a station x is known, then A, B, C, and D can be evaluated. Then, from Eqs. (9) and (10), dy_v/dx, dZ_v/dx, and hence dZ_v/dx can be obtained. That is, for a given value of x and Z_v, dZ_v/dx can be calculated. Hence we can write

$$\frac{dZ_v}{dx} = F(x, Z_v)$$

although the function is not explicit. This is just a first-order nonlinear equation in Z_v, and this has been solved by a fourth-order Runge-Kutta method. The starting solution was obtained from the solution for a delta wing of semivortex angle equal to the slope of the wing planform at its apex.

Conclusions

The method just described has been applied to a number of wing configurations. Results for a wing of planform, $s = \frac{2}{3}x - x^2/8$ (wing 2A of Ref. 3, span = 2.0, root chord = 6.0), are presented in Figs. 2 and 3.

It was seen in the case of delta wings[1] that the Brown and Michael method gives higher values for lift coefficients than experimental values. It was also observed[4] that the Z location of the vortex core is predicted quite well by this method, whereas the predicted y location is more toward the leading edge than the experimental observations. From Figs. 2 and 3, it can be seen that the same trend is repeated in the case of nonconical wings. (yv has been given only for $0 < x < 3.0$ in Ref. 5.) Although the Polhamus suction analogy method[3] predicts C_L quite accurately, it does not give any information about the vortex sheet. The results obtained from the Brown and Michael method could be used to extend Smith's vortex sheet model[4] to wings of nonconical planform.

References

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